REPORT DOCUMENTATION PAGE

AD-A280 450

Form Approved
OMB No. 0704-0188

1. AGENCY USE ONLY (Leave blank)	2. REPORT DATE	3. REPORT TYPE AND DATES COVERED
		Final Technical - 4/1/91-3/31/94

4. TITLE AND SUBTITLE	5. FUNDING NUMBERS
Design of Robust Controllers: Frequency Domain Methods and Their Non-linear extensions.	AFOSR-91-0166 2304/A1 61102F

6. AUTHOR(S)

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DTIC ELECTE JUN 21 1994 S B D

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)	8. PERFORMING ORGANIZATION REPORT NUMBER
Regents of the University of California University of California, San Diego 9500 Gilman Drive La Jolla, CA 92093-0934	AFOSR-91-0166 AFOSR-TR- 94 0359

9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSORING/MONITORING AGENCY REPORT NUMBER
Department of the Air Force Air Force Office of Scientific Research (AFSC) Bolling Air Force Base, Washington DC 20332-6448	

11. SUPPLEMENTARY NOTES

DTIC QUALITY INSPECTED 2

12a. DISTRIBUTION/AVAILABILITY STATEMENT	12b. DISTRIBUTION CODE
Approved for public release; distribution unlimited.	

13. ABSTRACT (Maximum 200 words)

The design of a system or circuit in which stability is a key constraint frequently leads to an optimization problem over the space of functions analytic on the right half plane (R.H.P.) Mathematical techniques for solving such optimization problems for mean square error (L^2 error) criteria have been widespread in engineering since the time of Wiener. Much of this research goes to developing techniques for handling worst case error (L^∞ error) criteria. These occur naturally in design of control systems and amplifiers. Practically speaking there is evidence that frequency domain L^∞ criteria control system designs have desirable robustness properties. The ultimate objective is to develop a new CAD approach to MIMO control design which has the flavor of classical control as well as a systematic approach to worst case frequency domain design as it occurs in many areas. The promise of this approach is sufficient to have attracted many investigators and it is currently the focus of much attention.

This research addresses many aspects of the problem. They range from the development of computer algorithms of a radically different type to the discovery of theoretical methods for understanding computational design. Also considerable progress was made in extending existing H^∞ control to nonlinear plants.

Another major effort involves computer algebra for systems research. The objective is to treat (on a computer) systems formulas of the type an investigator would manipulate by hand. Considerable software was developed along these lines.

14. SUBJECT TERMS	15. NUMBER OF PAGES
	16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT	18. SECURITY CLASSIFICATION OF THIS PAGE	19. SECURITY CLASSIFICATION OF ABSTRACT	20. LIMITATION OF ABSTRACT
UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED	UNCLASSIFIED

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)

H^{∞} CONTROL FOR NONLINEAR AND LINEAR SYSTEMS AFOSR -91-01-66

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FINAL REPORT

The research concerns H^{∞} control but focuses on substantially different parts of the subject, namely, nonlinear systems, optimization theory and algorithms for frequency domain design, and computer algebra tailored to systems and control research.

Nonlinear systems

The modern approach to worst case design in the frequency domain arose from studies of amplifier design the "dual" problem of making a circuit dissipative using feedback. For linear systems key cases of this were solved in 1965 (SISO) by Youla and Saito and (MIMO) in 1976 by Helton. In the early 80's Zames and Francis formulated H^{∞} control and solved the math problem by drawing on the earlier solutions to this circuits problem. In the beginning the subject of H^{∞} control evolved quickly in significant part because key math problems were already reasonably understood by operator theorists. I participated in this earlier work (e.g. solved the MIMO H^{∞} control problem with Zames and Francis, also Pearson and Chang) but at the same time begin pushing in new directions: nonlinear plants and an H^{∞} approach to classical control.

Of the various solutions to CTRL one which is easy to implement and numerically sound is the Doyle-Glover-Kargonekar-Francis DGKF two Riccati equation solution. Consequently extending this to nonlinear plants is of considerable importance. Through the last 3 years there has been considerable progress by Isidori and coworkers and by our group (Ball Helton Walker Zhan). Isidori et al find local sufficient conditions and compute (with Krener's software) power series solutions to model problems. All of these approaches assume something like the dimension of the compensator's state-space equals that of the controller state-space.

Our results [BHW] say:

Result 1 *The DGKF equations are 2 equations each in n dimensional space. These generalize to the nonlinear case as one Riccati P.D.E. in 2n variables. A positive solution to it is necessary for CTRL to have a solution. In the linear case the 2n dimensional Riccati P.D.E. easily yields the two DGKF equations.*

There is a surprisingly strong yet general separation theorem which limits the type of controller you can effectively use.

There is a formula which is reasonable to try for the controller. Only the input term for the controller is a compromise.

Wei Zhan and I analyzed completely those compensators whose state-space have the same dimension as the plant's state-space:

Result 2 *There is a strange type of equation which is a mixture of a first order P.D.E. and best approximation operators which we call a Tchebychev Riccati PDE. Existence of a positive solution to this equation is equivalent to a type of solution to CTRL.*

It is extremely unlikely that Tchebychev Riccati PDE will ever be solved exactly. However, now we know the enemy and this should help organize compromises in a systematic fashion.

Also there is progress on evaluating performance of piecewise linear systems. We took a typical architecture (a la Campo Morari) for a system with saturation and extracted one of the key computational difficulties. These systems are piecewise linear and continuous. Work in progress with Ball shows that a key object for a dissipative system , called a storage function, must be continuous. We then made a natural compromise. The continuity of the storage function forces constraints which make analyzing such systems not a Linear Matrix Inequality. We found a sequence

of steps which extracted the non LMI part and allowed one to solve the problem of determining performance of such systems by doing first an LMI check, then a side test then an LMI, etc.

There are still basic theoretical issues. In the last year James and Baras have necessary and sufficient conditions on the H^∞ control problem when one allows an infinite dimensional state-space. Under a saddle point assumption these reduce to one due to van der Schaft. Krener has results of a similar tone. James will visit here for most of the spring quarter.

A main open issue in the subject is finding compromise solutions to the equations produced above. Ultimately I see much of the field as consisting of sensible ways of finding conservative solutions to the equations which arise in the theoretical studies above.

Optimization over H^∞

Much of my effort goes to studying a basic question of worst case frequency domain design where stability of the system is the key constraint. This is the H^∞ optimization problem which is crucial in several branches of engineering.

The fundamental H^∞ problem of control.

First we state the core mathematics problem graphically. At each frequency ω we are given a set $\mathcal{S}_\omega(c) \subset \mathbf{C}^N$, called the *specification set*. The objective is to find a function T with no poles in the R.H.P. so that each $T(j\omega)$ belongs to $\mathcal{S}_\omega(c)$. In fact there is a simple picture to think of in connection with a design

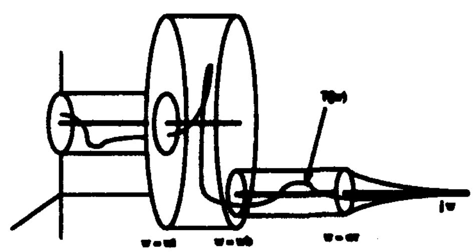

Figure 1

Typically there is a nested family of target sets $\mathcal{S}_\omega(c)$ parameterized by a performance level c; the smaller the sets the better the performance. For the *optimal* c a solution T exists but no solution exists for tighter specs.

The Horowitz templates of control can be transformed into this type of picture. When each $\mathcal{S}_\omega(c)$ is a "disk" this problem is solved by transformations of "classical

pure" mathematics done in the late 1970's by Helton. Many different solutions to this problem in many different coordinates were worked out by engineers in the last 15 years since it is the subject of H^∞ control. Competing constraints and plant uncertainty lead immediately to spec sets which are not disks.

The graphical problem of Fig2 can be formulated analytically in terms of a performance function Γ as

(OPT) Given a positive valued function Γ on $\mathbf{R} \times \mathbf{C}^N$ (which is a performance measure), find $\gamma^* \geq 0$ and f^* in A_N which solve

$$\gamma^* = \inf_{f \in A_N} \sup_{\omega} \Gamma(\omega, f(j\omega)) .$$

and this of course is what one puts in a computer. Collaborators and I have a very broad based attack on the problem which addresses most aspects of it.

From qualitative theory to numerical algorithms and diagnostics.

While little was known about this problem 10 years ago there has been a lot of progress, and now we have substantial theory. In progress is an elementary book with Merino on control system design which gives our methodology for setting design problems as formal optimization problems. Then our software solves the problems. The software runs under Mathematica and can be obtained from anopt@ucsd.edu.

We shall not sketch all that is known about OPT but mention one dramatic qualitative result (with D. Marshall)

Result 3 *For a "properly formulated" SISO control problem the optimal compensator is unique.*

Here no convexity is assumed.

Ironically one of the most practical results on an optimization problem is characterization of the optimum, since this is the basis for numerics. Our result is easier to state on the unit disk Δ and the unit circle $\mathbf{T}$ rather than on the R.H.P. and the $j\omega$-axis. Also we state it only for the N=2 MIMO case. Roughly the optimality condition for solutions to OPT is

Result 4 *Given* Γ *a smooth function. Necessary an sufficient for a smooth function* T^* *in* H^∞ *satisfying* $a(e^{i\theta}) = \frac{\partial \Gamma}{\partial z}(e^{i\theta}, T^*(e^{i\theta}))$ *is never* 0 *on* $\mathbf{T}$ *to be a local solution to* OPT *is*

I $\Gamma(e^{i\theta}, T^*(e^{i\theta}))$ *is constant in* $e^{i\theta}$.

II There exist F_1 *and* F_2 *analytic on the disk and* λ *a positive function on the circle such that for all* $e^{i\theta} \in \mathbf{T}$,

$$\frac{\partial \Gamma}{\partial z_1}(e^{i\theta}, T^*(e^{i\theta})) = e^{i\theta}\lambda(e^{i\theta})F_1(e^{i\theta})$$

$$\frac{\partial \Gamma}{\partial z_2}(e^{i\theta}, T^*(e^{i\theta})) = e^{i\theta}\lambda(e^{i\theta})F_2(e^{i\theta})$$

III A condition on second derivatives of Γ.

A typical computational strategy is to apply Newton's method to (I) and (II) above to solve them thereby solving OPT. Even when the spec sets were disks, subcases of which have been studied for 80 years, Newton's method was never successfully applied directly to this problem. The difficulty is that the problem is highly degenerate. However, recently O. Merino, T. Walker and I obtained

Result 5 *There is a functional analysis transform of (I) and (II) which yields nondegenerate equations and so Newton's method applied to them is second order convergent.*

Consequently we are finally obtaining satisfactory computational methods for solving OPT. In order to show this we give an example and optimize it using Newton iteration (Table 1) and the Disk iteration (Table 2).

EXAMPLE The problem is to solve OPT for the performance function

$$\Gamma(e^{i\theta}, z_1, z_2) = |z_1|^2+|z_2|^2+|100+e^{i\theta}z_1+.1(z_1+z_2+z_1z_2)|^2+|100+e^{i\theta}z_2+.1(z_1+z_2+z_1z_2)|^2$$

The (absolute) optimal value is $\gamma^* = 3800$, attained at two different points in function space: the constant functions $f_1^* = (30, -30)$ and $f_2^* = (-30, 30)$. There are no other local solutions to OPT.

The meaning of each column in Table 1 is as follows: It (Iteration number), Value (Current value of $\sup_\theta \Gamma(\cdot, f)$), || f - f* || (The true error, f* is the solution) , OT1 (checks for equation I of Result 4), OT2 (checks for equation II of Result 4), NED (A measure of numerical error).

The discretization of the problem is carried out by sampling functions on a grid of 256 equally spaced points on the unit circle. The Newton iteration is initialized at $f_0(e^{i\theta}) = (29.6 + .1e^{i\theta}, -30.4 - .0001e^{i\theta} + .001(e^{i\theta})^2)$, which is near a local solution. Observe how the diagnostics OT1 and OT2 tend to zero at essentially quadratic rate.

The same holds for the true error $\| f^k - f^* \|_{L_N^\infty}$. Compare Table 1 (Newton's method – quadratic rate of convergence) with Table 2 (previous method Disk iteration – linear rate).

Table 1: Newton iteration run

It :	Value SupGamma	:||fk-f*||:	Optimality Tests OT1	OT2	: Error NED
0	3.8165268771511E+03	5.7E-01	8.3E-03	1.6E+00	N/A
1	3.8009177903975E+03	5.3E-02	4.9E-04	3.9E-01	8. E-13
2	3.8000812563274E+03	2.7E-03	4. E-05	1.5E-02D	1.1E-11
3	3.8000001798719E+03	4.4E-06	6.3E-08	2. E-05C	1.9E-11
4	3.8000000000006E+03	1.5E-11	2.4E-13	3.6E-10D	5.3E-10

Table 2: Disk iteration method run

It :	Value SupGamma	:||fk-f*||:	Optimality Tests OT1	OT2	: Error NED
0	3.8165268771511E+03	5.7E-01	8.3E-03	1.6E+00	N/A
1	3.8014192462609E+03	1.4E-01	5.6E-04	1.1E+00D	3.1E+00
2	3.8004689130398E+03	1.3E-02	3. E-04	5.4E-02D	1.1E-02
3	3.8001160653124E+03	4.6E-03	6.2E-05	4. E-02C	1.1E-03
4	3.8000022236943E+03	3. E-03	9.8E-07	3.3E-02D	8.4E-05
5	3.8000007986885E+03	1.1E-03	4.5E-07	1.1E-02C	3.3E-06
6	3.8000001592016E+03	7.1E-04	1.2E-07	7.6E-03D	1.5E-06
7	3.8000000629256E+03	1.3E-04	3.7E-08	1.4E-03D	8.7E-07
8	3.8000000028458E+03	1.1E-04	1.9E-09	1.2E-03C	4.9E-05

Time domain constraints

Recently we were able to add time domain constraints to (OPT) and obtain optimality conditions extending Result 4 to this case. We consider a constrained optimization problem, named **Constr-OPT**, where the minimization is done over analytic functions (f_1, f_2) that satisfy a given set of constraints

$$\int_0^{2\pi} f_1 \overline{G_{1,\ell}} d\theta + \int_0^{2\pi} f_2 \overline{G_{2,\ell}} d\theta \geq 0, \qquad \ell = 1, \ldots, n$$

where the functions $G_{i,j}$ are analytic. We obtained,

Result 6 *Given Γ a smooth function and constraints as above. Necessary an sufficient for a smooth function T^* in H^∞ satisfying $a(e^{i\theta}) = \frac{\partial\Gamma}{\partial z}(e^{i\theta}, T^*(e^{i\theta}))$ is never 0 on $\mathbf{T}$ to be a local solution to* Constr-OPT *is*

I $\Gamma(e^{i\theta}, T^*(e^{i\theta}))$ is constant in $e^{i\theta}$.

II There exist F_1 and F_2 analytic on the disk, λ a positive function on the circle, and nonnegative constants $\kappa_1 \geq 0, \ldots, \kappa_n$ such that for all $e^{i\theta} \in \mathbf{T}$,

$$\frac{\partial\Gamma}{\partial z_1}(e^{i\theta}, T^*(e^{i\theta})) = \lambda(e^{i\theta})\left(e^{i\theta}F_1(e^{i\theta}) + \kappa_1\overline{G_{1,1}} + \ldots + \kappa_n\overline{G_{1,n}}\right)$$

$$\frac{\partial\Gamma}{\partial z_2}(e^{i\theta}, T^*(e^{i\theta})) = \lambda(e^{i\theta})\left(e^{i\theta}F_2(e^{i\theta}) + \kappa_1\overline{G_{2,1}} + \ldots + \kappa_n\overline{G_{2,n}}\right)$$

III A condition on second derivatives of Γ.

Further analysis shows that Results 4 and 6 mesh very well for the purpose of constructing computer algorithms. We have worked out such algorithms and began testing.

Of independent interest is that all of this represents a new connection between engineering and an existing branch of the mathematics area Several Complex Variables.

Computer algebra for systems research

There has been substantial work on computer algebra for engineering problems. For example, one specifies the systems or circuit components as letters say R_1 R_2 C_1 C_2 for resistor and capacitor values and the computer produces the formula for the transfer function (no matter how formidable). Then one can manipulate it on the computer.

Our approach is quite different. If one reads a typical article on A,B,C,D systems in the control transactions one finds that most of the algebra involved is non commutative rather than commutative. Thus for symbolic computing to have much impact on linear systems research one needs a program which will do noncommuting operations. Mathematica, Macsyma and Maple do not (contrary to what a salesman will tell you). For example, the most basic command

$$Expand[A**(B+C)]$$

gives $A*B + A*C$ if A, B, C commute but not if they do not. We have a package NCAlgebra which runs under Mathematica which does the basic operations, block matrix manipulations, and other things. The package might be seen as a competitor to a yellow pad. Like Mathematica the emphasis is on interaction with the program and flexibility.

Mins and maxes of hamiltonians Originally we wrote the package to do linear H^∞ control research. In particular, the main object in studying CTRL is the an energy balance (game theoretic) hamiltonian, For linear plants

$$\begin{aligned} H(x, z, W, \quad \nabla\varepsilon) = {} & \nabla_x\varepsilon(x,z)^T(Ax + B_1W + B_2cz) \\ & -W^TW + \|C_1x + D_{12}cz\|^2 + \nabla_z\varepsilon(x,z)^T[b(C_2x + D_{21}W) + az] \end{aligned}$$

In the notation of NCAlgebra it is

$$\begin{aligned} Ham \; &= tp[GEx[x,z]] ** (A ** x + B1 ** W + B2 ** c ** z) - tp[W] ** W \\ &(tp[x] ** tp[C1] + tp[z] ** tp[c] ** D12) ** (C1 ** x + D12 ** c ** z) \\ &+ tp[GEz[x,z]] ** (b ** (C2 ** x + D21 ** W) + a ** z) \end{aligned}$$

One must compute critical points (maxes or mins) of this in W, a, b, c in various orders which of course while routine is a tedious process. Also any variation on the problem produces a new hamiltonian and requires another tedious computation. NCAlgebra automates this. For example,

$$\begin{array}{ll} critW = Crit[Ham, W]; & HnoW = Ham/.critW; \\ critWc = Crit[HnoW, c]; & HnoWc = HnoW/.critWc; \end{array}$$

finds the critical point of Ham in W then in c and and evaluates Ham at these critical points. This same 4 lines applies to hamiltonians which arise in other control problems.

Simplification of messy formulas While NCAlgebra can be used as a yellow pad we are beginning to add serious automatic simplification commands. Wavrik, Stankus and I are now doing research in computer simplification for A, B, C, D type linear systems, in a highly noncommutative setting. The objective is in each particular situation to find a list of simplifying rules. A complete list of rules (called a Grobner basis GB) has the property that if it is applied to an expression until nothing changes then the expression is as simple as possible in a certain sense. Recently, Wavrik and I obtained

Result 7 *For the formulas which occur in studying energy conserving (lossless) systems. The GB while infinite can be summarized as a list of 32 rules some of which depend on an integer parameter. We give the list. It is a powerful tool for studying a particular class of systems. The list was discovered last year and actually proved (with Stankus) to be a GB very recently.*

A subset of these rules is now in a function NCSimplifyRational[expression] inside our NCAlgebra package. They are very effective on a limited class of expressions but even that makes them very useful. Now we have some experience in areas which use Lyapunov and Riccati equations and a line of experiments involving systems theory computations which explore them. It is becoming clear that this is tricky business and we are developing strategies to use computer algebra to obtain systems theory results. This is a matter of putting equations in a simplified canonical form. This area is wide open since in noncommuting situations the implications for linear systems theory are not explored.

Technology Transfer

We have two computer programs which run under Mathematica which are publically available.

NCAlgebra our non commuting algebra package has potential applications in many fields. Recently Mathematica's mathsource started distributing it and in fact they appear to be recommending it widely.

OPTDesign our classical control program is available from us (send request to *anopt@osiris.ucsd.edu*). We do not intend to start pushing it heavily until our book is finished, since this is the only account which ties everything together. Recently, we completed a major cleaning of the most basic version of Anopt,the optimization program underlying OPTDesign. This was to prepare for porting it to Matlab, which other groups have expressed an interest in doing.

Another level of transfer is from pure to applied mathematics. For example, in the last decade progress in H^∞ control was expedited by close connections with operator theorists who were originally in pure mathematics but who now work the mathematics of engineering systems. This originated with discoveries by DeWilde, Fuhrmann and I made in the early 1970's.

The work on optimization over analytic functions represents a new connection between engineering and an existing branch of several complex variables. Now little collaboration exists between workers in these areas. A bi-product of our development of (OPT) is possiblely that a new group of pure mathematicians will become interested in engineering.

Final Technical Report

P.I. Name: Helton, John W.
Institution: University of California, San Diego
Grant No.: AFOSR 91-0166

III. PUBLICATIONS

Articles that appeared in 1991

J. W. Helton, P. G. Spain and N. J. Young: Tracking Poles and Representing Hankel Operators Directly from Data, *Numerische Mathematik* 58 No. 6, (1991), 641-660.

J. W. Helton and O. Merino: "Optimal Analytic Disks," Collection: Several complex variables and complex geometry, Part 2, (Santa Cruz, CA, 1991) Proc. Sympos. Pure Math., 5:251-262.

F. J. Helton, J. M. Greene and J. W. Helton: A measure of control sensitivity and saturation effects for tokamak equilibria, *Plasma Physics and Controlled Fusion*, Vol. 33 No. 7, (1991), 827-845.

J. W. Helton, C. Foias, B. Frances, H. Kwakernaak, J. B. Pearson: H^∞-Control Theory, *Lecture notes in Math.*, Vol. 1496, (1991), pp. 1-222.

Articles that appeared in 1992

M. L. Walker and J. William Helton: High Frequency Inverse Scattering and the Luneberg-Kline Asymptotic Expansion, *IEEE Transactions Antennas and Propagation*, April (1992), Vol. 40, No. 4, 450-453.

J. A. Ball and J. William Helton: Nonlinear H^∞- Control Theory for Stable Plants, *J. Mathematics of Control, Signals, and Systems,* Springer-Verlag, New York, Inc., 1992, 5:233-261.

J. A. Ball and J. William Helton: Inner- Outer Factorization of Nonlinear Operators. *J. of Func. Analysis,* Vol. 104, No. 2, 1992, pp. 363-413.

J. W. Helton M. L. Walker and W. Zhan: H^∞- control using compensators with access to the command signals, 31st CDC, 1992, pp. 955-956.

J. W. Helton O. Merino and T. E. Walker: Conditions for optimality over H^∞: Numerical algorithms, 31st CDC, 1992, pp. 965-966.

J. A. Ball J. W. Helton, and M. L. Walker: Nonlinear H^∞- control and the bounded read lemma, 31st CDC, 1992, pp. 1045-1049.

J. A. Ball, J. William Helton and M. Verma: A factorization principle for stabilization of linear control systems, *Journal of Nonlinear and Robust Control* (1992), pp. 1-106.

Articles that appeared in 1993

NCAlgebra (Version 0.2) A Mathematica package for operator algebras and engineering systems (1993), (available from *ncalg@osiris.ucsd.edu*)

J. A. Ball, J. William Helton, and M. L. Walker: H^∞- Control for Nonlinear Systems with Output Feedback, *IEEE Transactions on Automatic Control*, Vol. 38, No. 4, April 1993. pp. 546-559.

J. W. Helton, and H. Woerdeman: Symmetric Hankel operators: minimal norm extensions and eigenstructures, *Linear Algebra and its Applications*, Vol 185, May 1993. pp. 1-19.

J. W. Helton, and O. Merino: A novel approach to accelerating Newton's method for sup-norm optimization arising in H^∞ - control, *Journal of Optimization Theory and Applications*, Vol. 78, No. 3, September 1993. pp. 553-578.

J.W. Helton; Merino, O.: Conditions for optimality over H^∞ *SIAM Journal on Control and Optimization*, vol.31, No.6. November 1993, pp. 1379-415.

J.W. Helton and O. Merino: H^∞ optimization with time domain and other constraints, 32nd CDC, December 1993, pp. 196-201.

J.W.Helton and Wei Zhan Piecewise Ricatti equations and the bounded real lemma, 32nd CDC, December 1993, pp. 196-201.

J. W. Helton, O. Merino, and T. E. Walker: Numerical optimization over spaces of analytic functions, *Indiana Journal of Math.* Vol 42, No.3, 1993.

Articles published/to be published 1994 - partial list

J.W. Helton; Lam D; and Woerdeman HJ: Sparsity patterns with high rank extremal positive semidefinite matrices, *Siam Journal on Matrix Analysis and Applications,* Vol.15, No. 1, January 1994. pp. 299-312.

J.W. Helton and Wei Zhan: A inequality governing nonlinear H^{∞} - control, *System Control Letters*, 1994. 22:157-165.

J. W. Helton, and J. Wavrik: Rules for Computer Simplification of the Formulas in Operator Theory and Linear Systems, to appear in *Operator Theory Advances and Applications,* 1994.

J. W. Helton;Bailey, F.N, and and O. Merino: Alternatiave Approaches in Frequency Domain Design of Single Loop Feedback Systems with Plant Uncertainty, to appear in 1994.

Final Technical Report

P.I. Name: Helton, John W.
Institution: University of California, San Diego
Grant No.: AFOSR 91-0166

IV. Researchers

Faculty:	J. William Helton
Postdocs:	Mark Stankus
	Wei Zhan
	Orlando Merino
	Matthew James
Graduate Students:	Trent Walker
	Julia Myers
	John Boyd
	Daniel Cunningham
Other:	Pablo Herrero (undergraduate student)
	Mike Lindelsee (undergraduate student)
	Stan Yoshinobu (undergraduate student)
	Sean Kelly (undergraduate student)
	Emmanuel Gamboa (undergraduate student)
	John Studaris (undergraduate student)

Final Technical Report

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V. Professional Talks & Presentation of Papers 1991-1994

Japan - June 1991
Workshop on operator theory.
Speaker and the US organizer.
Sapporo -- Hokiado Univ.

Japan - June 1991
Mathematical Theory of Networks and Systems conference:
Three talks.
Kobe

Israel- March 1992-sponsored by Binational Foundation
Conference on operator theory
Speaker
Beer Shev

Israel- March 1992-sponsored by Binational FoundationConference
on operator theory
Speaker
Tel Aviv

Southern Calif Partial Differential equations Conference.
April 10 -12, 1992
Speaker
University of California Santa Barbara

Great Plains Operator Theory Symposium - May 14 - 17, 1992
Pleniary address
University of Iowa

Non linear systems confence - May 29-30, 1992
Speaker

Washington University, St Louis ,Mo.

Tiawan-- sponsored by Tiawan Science Foundation - July 8-10, 1992
like a NSF regional conference in the US
7 hour lecture set
Tunghai Univ.

Conference on the Interface of Math and physics - July 12-14, 1992
Academica Sinica
Speaker
Taipei

Chinese University Hong Kong, July 1992
Talk canceled due to hurricane which shut down the city.
Hong Kong, CHINA

SIAM Control Conference On central organizing committee.
September 17 - 19, 1992
Institute Mathematical Applications
Raddison Hotel Minneapolis
Minneapolis, Minnesota

H. Widom's 60th birthday Conference , September 20 - 22, 1992
University of California, Santa Cruz
A principal speaker.
Santa Cruz, CA

In residence at Institute for Mathematical Applications,
October 7 -16, 1992
Special year in control.
Was invited to spend the year but I opted for 2 weeks.
Minneapolis, MN

Harvard Mathematics Department, October 26-29, 1992
Seminar
Harvard, MA

MIT October 26-29, 1992
Seminar
Cambridge, MA

Courant Institute of Mathematical Sciences, November 2, 1992
Colloquim

New York, NY

Virginia Polytechnic Mathematics Department
November 3 - 20, 1994
Seminar Math Dept
Blacksburg Virginia

Conference on Decision and Control, December 14-19, 1992
Presented papers with J. Ball; with T. Walker & O. Merino;
& with W. Zhan
Tuscon, Arizona

American Mathematical Society Meeting, January 1993
Special Session.
San Antonio, TX

Conference at Free University of Amsterdam July 1993
Speaker
Amsterdam, The Netherlands

Workshop: Operator Theory and Boundary Eigenvalue Problems
July 27 - 30, 1993
Speaker
Vienna Technical University
Wien, Austria

Int'l Symposium: Mathematical Theory of Networks and Systems
August 2 - 6, 1993
Pleniary Address
University of Regensburg

Workshop: Algebra and Networks, August 8 -15, 1993
Speaker
Val-Cenis, FRANCE

Conference on Decision Control, Decembr 1993
Presented papers with W. Zhan; & with O. Merino
San Antonio, TX

American Mathematical Society, January 1994
Special session speaker
Cincinnati, OH

THE REGENTS OF THE UNIVERSITY OF CALIFORNIA, SAN DIEGO

REPORT OF INVENTIONS AND SUBCONTRACTS

(Pursuant to "Patent Rights" Contract Clause) (See Instructions on Reverse Side.)

Form Approved
OMB No. 0704-0297
Expires Jun 30, 1992

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1a. NAME OF CONTRACTOR/SUBCONTRACTOR	c. CONTRACT NUMBER	2a. NAME OF GOVERNMENT PRIME CONTRACTOR	c. CONTRACT NUMBER	3. TYPE OF REPORT (X one)
		AFOSR	AFOSR-91-0166	a. INTERIM X b. FINAL
b. ADDRESS (Include ZIP Code)	d. AWARD DATE (YYMMDD)	b. ADDRESS (Include ZIP Code)	d. AWARD DATE (YYMMDD)	4. REPORTING PERIOD (YYMMDD)
La Jolla, CA 92093			91/4/1	a. FROM 91/4/1 b. TO 94/3/31

SECTION I – SUBJECT INVENTIONS

5. "SUBJECT INVENTIONS" REQUIRED TO BE REPORTED BY CONTRACTOR/SUBCONTRACTOR (If "None," so state)

a. NAME(S) OF INVENTOR(S) (Last, First, MI)	b. TITLE OF INVENTION(S)	c. DISCLOSURE NO., PATENT APPLICATION SERIAL NO. OR PATENT NO.	d. ELECTION TO FILE PATENT APPLICATIONS (1) United States (a) Yes	(1) United States (b) No	(2) Foreign (a) Yes	(2) Foreign (b) No	e. CONFIRMATORY INSTRUMENT OR ASSIGNMENT FORWARDED TO CONTRACTING OFFICER (1) Yes	(2) No
NONE	NONE							

8. EMPLOYER OF INVENTOR(S) NOT EMPLOYED BY CONTRACTOR/SUBCONTRACTOR		9. ELECTED FOREIGN COUNTRIES IN WHICH A PATENT APPLICATION WILL BE FILED	
(1)(a) Name of Inventor (Last, First, MI)	(2)(a) Name of Inventor (Last, First, MI)	(1) Title of Invention	(2) Foreign Countries of Patent Application
(b) Name of Employer	(b) Name of Employer		
(c) Address of Employer (Include ZIP Code)	(c) Address of Employer (Include ZIP Code)		

SECTION II – SUBCONTRACTS (Containing a "Patent Rights" clause)

6. SUBCONTRACTS AWARDED BY CONTRACTOR/SUBCONTRACTOR (If "None," so state)

a. NAME OF SUBCONTRACTOR(S)	b. ADDRESS (Include ZIP Code)	c. SUBCONTRACT NO.(S)	d. DFAR "PATENT RIGHTS" (1) Clause Number	(2) Date (YYMM)	e. DESCRIPTION OF WORK TO BE PERFORMED UNDER SUBCONTRACT(S)	f. SUBCONTRACT DATES (YYMMDD) (1) Award	(2) Estimated Completion
NONE	NONE						

SECTION III – CERTIFICATION

7. CERTIFICATION OF REPORT BY CONTRACTOR/SUBCONTRACTOR (Not required if [ ] Small Business or [ ] Non-Profit organization.) (X appropriate box)

a. NAME OF AUTHORIZED CONTRACTOR/SUBCONTRACTOR OFFICIAL (Last, First, MI) Bettye H. Albritton	c. I certify that the reporting party has procedures for prompt identification and timely disclosure of "Subject Inventions," that such procedures have been followed and that all "Subject Inventions" have been reported.
b. TITLE Contract and Grant Officer	d. SIGNATURE Dr. J. William Helton J W Helton e. DATE SIGNED May 19/94

DD Form 882, OCT 89

Previous editions are obsolete.